



**JSS COLLEGE OF ARTS, COMMERCE AND SCIENCE**

**(Autonomous)**

**B N ROAD, MYSURU- 570 025**

**DEPARTMENT OF MATHEMATICS**

**Syllabus**

**CHOICE BASED CREDIT SYSTEM**

**For B.Sc programmes**

- **Physics, Mathematics and Chemistry**
- **Physics, Mathematics and Computer Science**
- **Physics, Mathematics and Computer Maintenance**
- **Physics, Mathematics and Electronics**

**2019-2020**

**JSS COLLEGE OF ARTS, COMMERCE & SCIENCE  
(AUTONOMOUS)  
B.N ROAD, MYSORE  
DEPARTMENT OF MATHEMATICS  
B.Sc Syllabus (CBCS)  
LIST OF COURSES WITH CREDIT PATTREN**

Year	Semester	DSC/DSE/SEC	Title of the paper	Lecture +Practical hours per week	No. of Credits			Total Credits	Total hours		Maximum Marks in Exam/Assesment				Exam duration
					L	T	P		Th	Pr	Exam	IA		Total	
												C1	C2		
I B.Sc	I	DSC-I Theory	Differential Calculus	04	4	0	0	6	60	60	70	15	15	100	3h
		DSC-I Practical		04	0	0	2				35	7.5	7.5	50	
	II	DSC-II Theory	Differential Equations	04	4	0	0	6	60	60	70	15	15	100	3h
		DSC-II Practical		04	0	0	2				35	7.5	7.5	50	
II B.Sc	III	DSC-III Theory	Real Analysis	04	4	0	0	6	60	60	70	15	15	100	3h
		DSC-III Practical		04	0	0	2				35	7.5	7.5	50	
	IV	DSC-IV Theory	Algebra	04	4	0	0	6	60	60	70	15	15	100	3h
		DSC-IV Practical		04	0	0	2				35	7.5	7.5	50	
III B.Sc	V	DSE-I Theory	Linear Algebra / Matrices	04	4	0	0	6	60	60	70	15	15	100	3h
		DSE-I Practical		04	0	0	2				35	7.5	7.5	50	
	VI	DSE-II Theory	Complex Analysis / Numerical Methods	04	4	0	0	6	60	60	70	15	15	100	3h
		DSE-II Practical		04	0	0	2				35	7.5	7.5	50	
		SEC	Vector calculus	02	2	0	0				2	30	-	35	

**Scheme of Assessment:**

Credits L:T:P	Percentage			Maximum marks in the Exam /Assessment			Exam Duration	
	Th	Pr	IA	Th	Pr	IA	Th	Pr
4:0:2	50	20	30	70	70	30	3h	3h
5:1:0	70	-	30	70	-	30	3h	-
4:0:0	70	-	30	70	-	30	3h	-
2:0:2	35	35	30	50	70	30	2h	3h
3:0:0	70	-	30	70	-	30	3h	-
2:1:0	70	-	30	70	-	30	3h	-
2:0:0	70	-	30	50	-	30	2h	-
0:0:1	-	70	30	-	70	30	-	2h

**Note:** L-Lecture, T-Tutorial, P-Practical; Th- Theory, Pr-Practical,

I A- Internal Assessment

## SEMESTER - I

Credits: L: T: P = 4:0:0

Teaching hours: 4 hours per week

### DSC-I : Differential Calculus

**Unit I :** Limit and Continuity ( $\epsilon$  and  $\delta$  definition), Types of discontinuities, Differentiability of functions, Successive differentiation, Leibnitz's theorem, Partial differentiation, Euler's theorem on homogeneous functions.

**Unit II:** Linear Approximation theorem, Tangents and normals, Monotone functions, Maxima and Minima, Curvature, Radius of curvature, Centre of curvature, Evolutes

**Unit III :** Rolle's theorem, Mean Value theorems, Taylor's theorem with Lagrange's and Cauchy's forms of remainder, Taylor's series, Maclaurin's series of  $\sin x$ ,  $\cos x$ ,  $e^x$ ,  $\log(1+x)$ ,  $(1+x)^m$ , Maxima and Minima, Indeterminate forms.

**Unit IV:** Asymptotes, Envelopes, Singular points – Multiple points – Cusp, Node and conjugate points, Orthogonal Trajectories, Tracing of curves,

#### Reference Books:

1. Frank Ayres and Elliott Mendelson, Schaum's Outline of Calculus, 5th ed. USA: Mc. Graw Hill., 2008.
2. G B Thomas and R L Finney, Calculus and analytical geometry, Addison Wesley, 1995.
3. H. Anton, I. Birens and S. Davis, Calculus, John Wiley and Sons, Inc., 2002
4. J Edwards, An elementary treatise on the differential calculus: with Applications and numerous example, Reprint. Charleston, USA Biblio Bazaar, 2010.
5. Lipman Bers – Calculus, Volumes 1 and 2
6. N. Piskunov – Differential and Integral Calculus
7. N P Bali, Differential Calculus, India: Laxmi Publications (P) Ltd., 2010.
8. Serge Lang – First Course in Calculus
9. S Narayanan & T. K. Manicavachogam Pillay, Calculus.: S. Viswanathan Pvt. Ltd., vol. I & II 1996.
10. Shanthi Narayan and P K Mittal, Differential Calculus, Reprint. New Delhi: S Chand and Co. Pvt. Ltd., 2014.

**PRACTICAL COMPONENT-1**

**Credits: L : T: P = 0:0:2**

1. Introduction to Scilab.
2. Operators; trigonometric, inverse trigonometric functions in scilab.
3. Plotting of standard Cartesian curves using Scilab.
4. Plotting of standard polar curves using Scilab.
5. Plotting of standard parametric curves using Scilab.
6. Introduction to Maxima.
7. Creating variables, functions.
8. Creating a Maxima program (simple examples, loops, control sequence).
9. Differentiation and integration using maxima inbuilt functions.
10. Plotting of standard curves- Cartesian, Polar using Maxima.
11. Plotting of standard parametric curves using Maxima.
12. Geometrical meaning of Rolle's theorem of the functions on the given interval.
13. To verify Rolle's theorem , Lagrange's theorem and cauchy's mean value theorem
14. Finding Taylor's theorem for a given function.
15. To illustrate left hand and right hand limits for discontinuous functions.
16. To illustrate continuity of a function.
17. To illustrate differentiability of a function.

## SEMESTER-II

Credits: L: T: P = 4:0:0

Teaching hours: 4 hours per week

### DSC II: Differential Equations

**Unit I:** Linear differential equations of First order, Separation of variables, Equations with homogeneous coefficients, Exact differential equations, Linear differential equations of the form  $\frac{dy}{dx} + Py = Q$ , Integrating factors, rules to find an integrating factor, Bernoulli's Equations, Equations with coefficients linear in x and y.

**Unit II:** First order higher degree equations solvable for x, y, p, Clairaut's form. Methods for solving higher-order differential equations. Basic theory of linear differential equations, Wronskian, and its properties. Solving a differential equation by reducing its order, Simultaneous differential equations and Total differential equations.

**Unit III:** Linear homogenous equations with constant coefficients, Linear non-homogenous equations, The method of variation of parameters, Exact equations, Inverse Differential operators, The Cauchy-Euler equation.

**Unit IV:** Order and degree of partial differential equations, Concept of linear and non-linear partial differential equations, Formation of first order partial differential equations, Linear partial differential equation of first order, Lagrange's method, Charpit's method.

#### Reference Books:

1. Daniel A Murray – Introductory Course to Differential equations  
Earl David Rainville and Philip Edward Bedient – A short course in Differential equations, Prentice Hall College Div; 6th edition.
2. M D Raisinghania, Advanced Differential Equations, S Chand and Co. Pvt. Ltd., 2013.  
F. Ayres, Schaum's outline of theory and problems of Differential Equations, 1st ed. USA McGraw-Hill, 2010
3. S Narayanan and T K Manicavachogam Pillay, Differential Equations .: S V Publishers Private Ltd., 1981.
4. G F Simmons, Differential equation with Applications and historical notes, 2nd ed.: McGraw-Hill Publishing Company, Oct 1991.
5. G. Stephenson – An introduction to Partial Differential Equations.
6. B. S. Grewal – Higher Engineering Mathematics  
E. Kreyszig – Advanced Engineering Mathematics
7. E. D. Rainville and P E Bedient – A Short Course in Differential Equations
8. D. A Murray – Introductory Course in Differential Equations.
9. G. F. Simmons – Differential Equations with Applications and Historical notes.
10. F. Ayres – Differential Equations (Schaum Series)
11. Martin Brown – Application of Differential Equations.
12. Shepley L. Ross, Differential Equations, 3<sup>rd</sup> Ed, John Wiley and Sons, 1984.

**PRACTICAL COMPONENTS-II**

**Credits: L:T: P = 0:0:2**

1. Obtaining partial derivatives of some standard functions
2. Solution of Differential equation and plotting the solution-I
3. Solution of Differential equation and plotting the solution-II
4. Solution of Differential equation and plotting the solution-III
5. Solution of Differential equation and plotting the solution-IV
6. Finding complementary function and particular integral of constant coefficient second and higher order ordinary differential equations.
7. Solving second order linear partial differential equations in two variables with constant coefficient.
8. Solutions to the problems on total and simultaneous differential equations.
9. Solutions to the problems on different types of partial differential equations.
10. Solution of Cauchy problem for first order partial differential equation.
11. Plotting the characteristics for the first order partial differential equation.
12. Plot the integral surfaces of a given first order partial differential equation with initial data.

### SEMESTER III

Credits: L: T: P = 4:0:0

Teaching hours: 4 hours per week

#### DSC III : Real Analysis

**Unit I:** Finite and infinite sets, examples of countable and uncountable sets. Real line, bounded sets, supremum and infimum, completeness property of  $\mathbb{R}$ , Archimedean property of  $\mathbb{R}$ , intervals. Concept of cluster points and statement of Bolzano-Weierstrass theorem.

**Unit II:** Real Sequence, Bounded sequence, Cauchy convergence criterion for sequences. Cauchy's theorem on limits, order preservation and squeeze theorem, monotone sequences and their convergence .

**Unit III:** Infinite series. Cauchy convergence criterion for series, positive term series, geometric series, comparison test, convergence of p-series, Root test, Ratio test, alternating series, Leibnitz's test , Definition and examples of absolute and conditional convergence.

**Unit IV:** Sequences and series of functions, Point wise and uniform convergence. Mn-test, M-test, Statements of the results about uniform convergence, Power series and radius of convergence.

#### Reference Books:

1. S.C Malik –Real Analysis
2. S.C.Malik and Savita Arora, *Mathematical Analysis*, 2nd ed. New Delhi, India: New Age international (P) Ltd., 1992
3. Richard R Goldberg, *Methods of Real Analysis*, Indian ed.
4. Asha Rani Singhal and M .K Singhal, *A first course in Real Analysis*
5. E.Kreyszig- *Advanced Engineering Mathematics*, Wiley India Pvt. Ltd.
6. Raisinghania M. D., *Laplace and Fourier Transforms* S. Chand publications.
7. *Principles of Mathematical Analysis*- Walter Rudin.
8. *Mathematical Analysis*- Tom M Apostol



**PRACTICAL COMPONENTS –III**

**Credits: L: T: P = 0:0:2**

1. Illustration of convergent, divergent and oscillatory sequences.
2. Plotting of recursive sequences.
3. Study of convergence of sequences through plotting
4. Illustration of convergent, divergent and oscillatory series.
5. To study the convergence and divergence of infinite series by plotting their sequences of partial sums.
6. Using Cauchy's criterion on the sequence of partial sums of the series to determine convergence of series.
7. Cauchy's root test by plotting  $n^{\text{th}}$  roots.
8. Ratio test by plotting the ratio of  $n^{\text{th}}$  and  $(n+1)^{\text{th}}$  terms.
9. Testing the convergence of binomial, exponential and logarithmic series and finding the sum.
10. To find the sum of the series and its radius of convergence.

## SEMESTER IV

Credits: L: T: P = 4:0:0

Teaching hours: 4 hours per week

### DSC IV: Algebra

**Unit I:** Definition and examples of groups, examples of abelian and non-abelian groups, the group  $Z_n$  of integers under addition modulo  $n$  and the group  $U(n)$  of units under multiplication modulo  $n$ . Cyclic groups from number systems, complex roots of unity, cyclic group, groups of symmetries, the permutation group, Group of quaternion's.

**Unit II:** Subgroups, cyclic subgroups, the concept of a subgroup generated by a subset and the commutator subgroup of group, examples of subgroups including the center of a group. Cosets, Index of subgroup, Lagrange's theorem, order of an element, Normal subgroups: their definition, examples, and characterizations, Quotient groups. Homomorphism, Kernel and Image, Isomorphism, Fundamental Theorem of Homomorphism.

**Unit III:** Definition and examples of rings, examples of commutative and non-commutative rings: rings from number systems,  $Z_n$  the ring of integers modulo  $n$ , ring of real quaternion's, rings of matrices, polynomial rings, and rings of continuous functions. Sub rings and ideals.

**Unit IV:** Integral domains and fields, examples of fields:  $Z_p$ ,  $Q$ ,  $R$ , and  $C$ . Field of rational functions. Homomorphisms, Isomorphism.

#### Reference Books :

1. I. N. Herstein – Topics in Algebra.
2. Joseph Gallian – Contemporary Abstract Algebra, Narosa Publishing House, New Delhi, Fourth Edition.
3. G. D. Birkhoff and S MacLane – A brief Survey of Modern Algebra.
4. J B Fraleigh – A first course in Abstract Algebra.
5. Michael Artin – Algebra, 2nd ed. New Delhi, India: PHI Learning Pvt. Ltd., 2011.
6. Vashista, A First Course in Modern Algebra, 11th ed.: Krishna Prakasan Mandir, 1980.
7. R Balakrishnan and N.Ramabadran, A Textbook of Modern Algebra, 1st ed. New Delhi, India: Vikas publishing house pvt. Ltd., 1991.
8. University algebra by N.S.Gopalakrishnan

**PRACTICAL COMPONENTS-IV**

**Credits: L: T: P = 0:0:2**

1. Verifying whether a given operator is binary or not.
2. To find identity element of a group.
3. To find inverse element of a group.
4. Finding all possible subgroups of a finite group.
5. Examples to verify Lagrange's theorem.
6. Illustrating homomorphism and isomorphism of groups.
7. Verification of normality of a given subgroup.
8. Verifying Cayley's theorem and isomorphism theorems.
9. Examples for finding left and right coset and finding the index of a group.
10. Examples on different types of rings.
11. Examples on integral domains and fields.
12. Examples on subrings, ideals and subrings which are not ideals.
13. Homomorphism and isomorphism of rings – illustrative examples.
14. Solving polynomial equations.
15. Finding the G.C.D of polynomials.
16. Finding units and associates.
17. Test for rational roots.

## SEMESTER V

**Credits: L: T: P = 4:0:0**

**Teaching hours: 4 hours per week**

### **DSE I: Linear Algebra**

**Unit I:** Vector spaces, subspaces, algebra of subspaces, quotient spaces, linear combination of vectors, linear span, linear independence, basis and dimension, dimension of subspaces, Direct sum of two subspaces, Quotient space.

**Unit II:** Inner Product, Inner Product of any two vectors in  $V(\mathbb{R})$ , Euclidean Vectorspace, Orthogonal Vectors, Ortho normal Basis, Orthogonal Projection, Orthogonal Compliment.

**Unit III:** Linear transformations, algebra of linear transformations, matrix representation of a linear transformation, null space, range, rank and nullity of a linear transformation,

**Unit IV:** Eigen values and Eigen vectors, Characteristic Polynomial. Isomorphism, Auto morphism, theorems, invertibility of non singular linear transformation, change of Basis and similar matrices.

#### Reference Books:

1. I. N. Herstein – Topics in Algebra.
2. Stewart – Introduction to Linear Algebra
3. S. Kumaresan – Linear Algebra
4. G. D. Birkhoff and S MacLane – A brief Survey of Modern Algebra.
5. N.S.Gopalakrishna – University Algebra
6. Saymour Lipschitz – Theory and Problems of Linear Algebra.
7. B.S Grewal – Higher engineering mathematics.
8. E.Kreyszig – Advanced Engineering Mathematics, Wiley India Pvt. Ltd.
9. J B Fraleigh – A first course in Abstract Algebra.

**PRACTICAL COMPONENTS–V**

**Credits: L: T: P = 0:0:2**

1. Vector space, subspace – illustrative examples.
2. Expressing a vector as a linear combination of given set of vectors.
3. Examples on linear dependence and independence of vectors.
4. Basis and Dimension – illustrative examples.
5. Verifying whether a given transformation is linear.
6. Finding matrix of a linear transformation.
7. Problems on rank and nullity.
8. Find characteristics polynomials.
9. To find Eigen values and their multiplicity.
10. Calculation of Eigen vector.
11. Change of basis.
12. Linear transformations to matrices and vice versa.
13. Matrix with respect to change of basis.
14. Orthogonal and orthonormal sets.
15. Gram- Schmidt orthogonalisation of the columns.

## SEMESTER V

**Credits: L: T: P = 4:0:0**

**Teaching hours: 4 hours per week**

### **DSE I A: Matrices**

**Unit I:** Types of matrices. Rank of a matrix. Invariance of rank under elementary transformations. Reduction to normal form, Solutions of linear homogeneous and non-homogeneous equations with number of equations and unknowns upto four.

**Unit II:** Matrices in diagonal form. Reduction to diagonal form upto matrices of order 3. Computation of matrix inverses using elementary row operations. Rank of matrix. Solutions of a system of linear equations using matrices. Illustrative examples of above concepts from Geometry, Physics, Chemistry, Combinatorics and Statistics.

**Unit III:**  $R_1, R_2, R_3$  as vector spaces over  $R$ . Standard basis for each of them. Concept of Linear Independence and examples of different bases. Subspaces of  $R_2, R_3$ .

**Unit IV:** Translation, Dilation, Rotation, Reflection in a point, line and plane. Matrix form of basic geometric transformations. Interpretation of eigen values and eigen vectors for such transformations and eigen spaces as invariant subspaces.

#### Reference Books:

1. A.I. Kostrikin, Introduction to Algebra, Springer Verlag, 1984.
2. S. H. Friedberg, A. L. Insel and L. E. Spence, Linear Algebra, Prentice Hall of India Pvt. Ltd., New Delhi, 2004.
3. Richard Bronson, Theory and Problems of Matrix Operations, Tata McGraw Hill, 1989.
4. E.Kreyszig – Advanced Engineering Mathematics, Wiely India Pvt. Ltd
5. B.S Grewal – Higher engineering mathematics

### **PRACTICAL COMPONENTS-V**

**Credits: L:T:P = 0:0:2**

1. Introduction to matrices and commands connected to the matrices.
2. Addition and subtraction of matrices.
3. Multiplication and transpose of matrices.
4. Power of a matrix.
5. Row reduced echelon form.
6. Rank of a matrix.
7. Adjoint of a matrix.
8. Inverse of a non-singular matrix.
9. Systems of linear equations.

10. Trace of a matrix.

## SEMESTER VI

Credits: L:T: P = 4:0:0

Teaching hours: 4 hours per week

### DSE II: Complex Analysis

**Unit I:** Complex numbers, Polar and exponential form of complex numbers, Triangular inequality, Geometry of complex numbers, Equations of lines and circles in complex form, Functions of complex variables, Limits, Limits involving the point at infinity, continuity. Properties of complex numbers, regions in the complex plane, Derivatives, differentiation formulas, Cauchy-Riemann equations, sufficient conditions for differentiability.

**Unit II:** Analytic functions, examples of analytic functions, exponential function, Logarithmic function, trigonometric function, Harmonic functions, Construction of Analytic functions.

**Unit III:** Definite integrals of functions. Contours, Contour integrals and its examples, upper bounds for moduli of contour integrals. Cauchy-Goursat theorem, Cauchy integral formula, Cauchy's inequality, Liouville's theorem and the fundamental theorem of algebra. Convergence of sequences and series.

**Unit IV:** Linear and Bilinear Transformations, Cross ratio of four points, Conformal mapping, Transformations of the form  $z^2$ ,  $\frac{1}{z}$ ,  $\sin z$ ,  $\cos z$ ,  $e^z$ ,  $\sinh z$ ,  $\cosh z$  etc, Laurent series and its examples, Poles and residues.

#### Reference Books:

1. L. V. Ahlfors – Complex Analysis
2. Bruce P. Palica – Introduction to the Theory of Function of a Complex Variable
3. Serge Lang – Complex Analysis
4. Shanthinarayan – Theory of Functions of a Complex Variable
5. S. Ponnuswamy – Foundations of Complex Analysis
6. R. P. Boas – Invitation to Complex Analysis.
7. R V Churchill & J W Brown, Complex Variables and Applications, 5th ed.: McGraw Hill Companies., 1989.
8. A R Vashista, Complex Analysis, Krishna Prakashana Mandir, 2012.

## PRACTICAL COMPONENTS-VI

Credits: L:T:P = 0:0:2

1. Declaring a complex number and graphical representation.
2. Complex numbers and their representations, operations like addition, multiplication, division, modulus, graphical representations of polar form.
3. To plot the complex functions and analyze the graph  
(i)  $f(z) = z$ , (ii)  $f(z) = z^3$ , (iii)  $f(z) = (z^4 - 1)^{1/4}$
4. Some problems on Cauchy – Riemann equations (polar forms).
5. Implementation of Milne – Thomson method of constructing analytic functions (simple examples).
6. Illustrating orthogonality of the surfaces obtained from the real and imaginary parts of an analytic function.
7. Verifying real and imaginary parts of an analytic function being harmonic (in polar coordinates)
8. Examples connected with Cauchy's integral theorem.
9. To compute the poles and corresponding residues of complex functions.
10. Illustrating the angle preserving property in a transformation.
11. Illustrating the circles are transformed to circles by a bilinear transformation.
12. To perform conformal mapping and bilinear transformations.



## SEMESTER VI

Credits: L:T: P = 4:0:0

Teaching hours: 4 hours per week

### DSE II B: Numerical Methods

**Unit I:** Algorithms, Convergence, Bisection method, False position method, Fixed point iteration method, Newton's method, Secant method, LU decomposition, Gauss-Jacobi, Gauss-Siedel and SOR iterative methods.

**Unit II:** Lagrange and Newton interpolation: linear and higher order, finite difference operators. Numerical differentiation: forward difference, backward difference and central Difference. Integration: trapezoidal rule, Simpson's rule, Euler's method.

#### Reference Books:

1. B. Bradie, A Friendly Introduction to Numerical Analysis, Pearson Education, India, 2007.
2. M.K. Jain, S.R.K. Iyengar and R.K. Jain, Numerical Methods for Scientific and Engineering Computation, 5th Ed., New age International Publisher, India, 2007.
3. Introduction to Numerical analysis by S.S.Shastri
4. B.S Grewal – Higher engineering mathematics
5. E.Kreyszig – Advanced Engineering Mathematics, Wiely India Pvt. Ltd

### PRACTICAL COMPONENTS-VI A

Credits: L:T:P = 0:0:2

1. Newton Gregory forward interpolation.
2. Lagrange interpolation.
3. Simpson's one-third method.
4. Simpson's three-eighth method.
5. Bisection method.
6. Regula-Falsi method.
7. Newton-Raphson method.
8. Modified Euler's metod.
9. Runge Kutta second order method.
10. Runge Kutta fourth order method.

## Vector Calculus

**Credits: L:T: P = 2:0:0**

**Teaching hours: 2 hours per week**

### **Skill Enhancement Course (SEC - II)**

**Unit I:** Differentiation and partial differentiation of a vector function. Derivative of sum, dot product and cross product of two vectors.

**Unit II:** Gradient, divergence and curl, Standard derivations and Exercise ,

### Reference Books:

1. Murray R Spiegel – Theory and problems of vector calculus.
2. Shanthinarayan and J N Kapur – A text book of Vector calculus.
3. B.S Grewal – Higher engineering mathematics.
4. Shanthi Narayan and P K Mittal, Differential Calculus, Reprint. New Delhi: S Chand and Co. Pvt. Ltd., 2014.

**Question Paper Pattern**

**Mathematics**

**Time:** 3 Hours

**Max. Marks:** 70

**Section – A**

**I. Answer any five questions.**

**5 x 2 = 10**

- 1.
- 2.
- 3.
- 4.
- 5.
- 6.
- 7.
- 8.

**Section – B**

**II. Answer any three questions.**

**3 x 5 = 15**

- 1.
- 2.
- 3.
- 4.
- 5.

**III. Answer any three questions.**

**3 x 5 = 15**

- 1.
- 2.
- 3.
- 4.
- 5.

**IV. Answer any three questions.**

**3 x 5 = 15**

- 1.
- 2.
- 3.
- 4.
- 5.

**V. Answer any three questions.**

**3 x 5 = 15**

- 1.
- 2.
- 3.
- 4.
- 5.